



1. (a) Express  $\frac{2}{(2r+1)(2r+3)}$  in partial fractions. (2)

(b) Using your answer to (a), find, in terms of  $n$ ,

$$\sum_{r=1}^n \frac{3}{(2r+1)(2r+3)}$$

Give your answer as a single fraction in its simplest form. (3)

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3. 
$$\frac{d^2y}{dx^2} + 4y - \sin x = 0$$

Given that  $y = \frac{1}{2}$  and  $\frac{dy}{dx} = \frac{1}{8}$  at  $x = 0$ ,

find a series expansion for  $y$  in terms of  $x$ , up to and including the term in  $x^3$ .

**(5)**

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**Question 5 continued**

Lined area for writing the answer to Question 5 continued.





6. (a) Use algebra to find the exact solutions of the equation

$$|2x^2 + 6x - 5| = 5 - 2x \quad (6)$$

(b) On the same diagram, sketch the curve with equation  $y = |2x^2 + 6x - 5|$  and the line with equation  $y = 5 - 2x$ , showing the  $x$ -coordinates of the points where the line crosses the curve. (3)

(c) Find the set of values of  $x$  for which

$$|2x^2 + 6x - 5| > 5 - 2x \quad (3)$$

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7. (a) Show that the transformation  $y = xv$  transforms the equation

$$4x^2 \frac{d^2y}{dx^2} - 8x \frac{dy}{dx} + (8 + 4x^2)y = x^4 \quad (I)$$

into the equation

$$4 \frac{d^2v}{dx^2} + 4v = x \quad (II) \quad (6)$$

(b) Solve the differential equation (II) to find  $v$  as a function of  $x$ .

(6)

(c) Hence state the general solution of the differential equation (I).

(1)

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8.

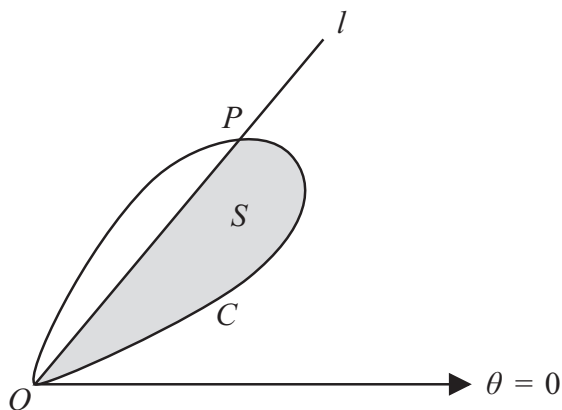


Figure 1

Figure 1 shows a curve  $C$  with polar equation  $r = a \sin 2\theta$ ,  $0 \leq \theta \leq \frac{\pi}{2}$ , and a half-line  $l$ .

The half-line  $l$  meets  $C$  at the pole  $O$  and at the point  $P$ . The tangent to  $C$  at  $P$  is parallel to the initial line. The polar coordinates of  $P$  are  $(R, \phi)$ .

(a) Show that  $\cos \phi = \frac{1}{\sqrt{3}}$  (6)

(b) Find the exact value of  $R$ . (2)

The region  $S$ , shown shaded in Figure 1, is bounded by  $C$  and  $l$ .

(c) Use calculus to show that the exact area of  $S$  is

$$\frac{1}{36} a^2 \left( 9 \arccos \left( \frac{1}{\sqrt{3}} \right) + \sqrt{2} \right)$$
(7)

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